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MATHS METHODS

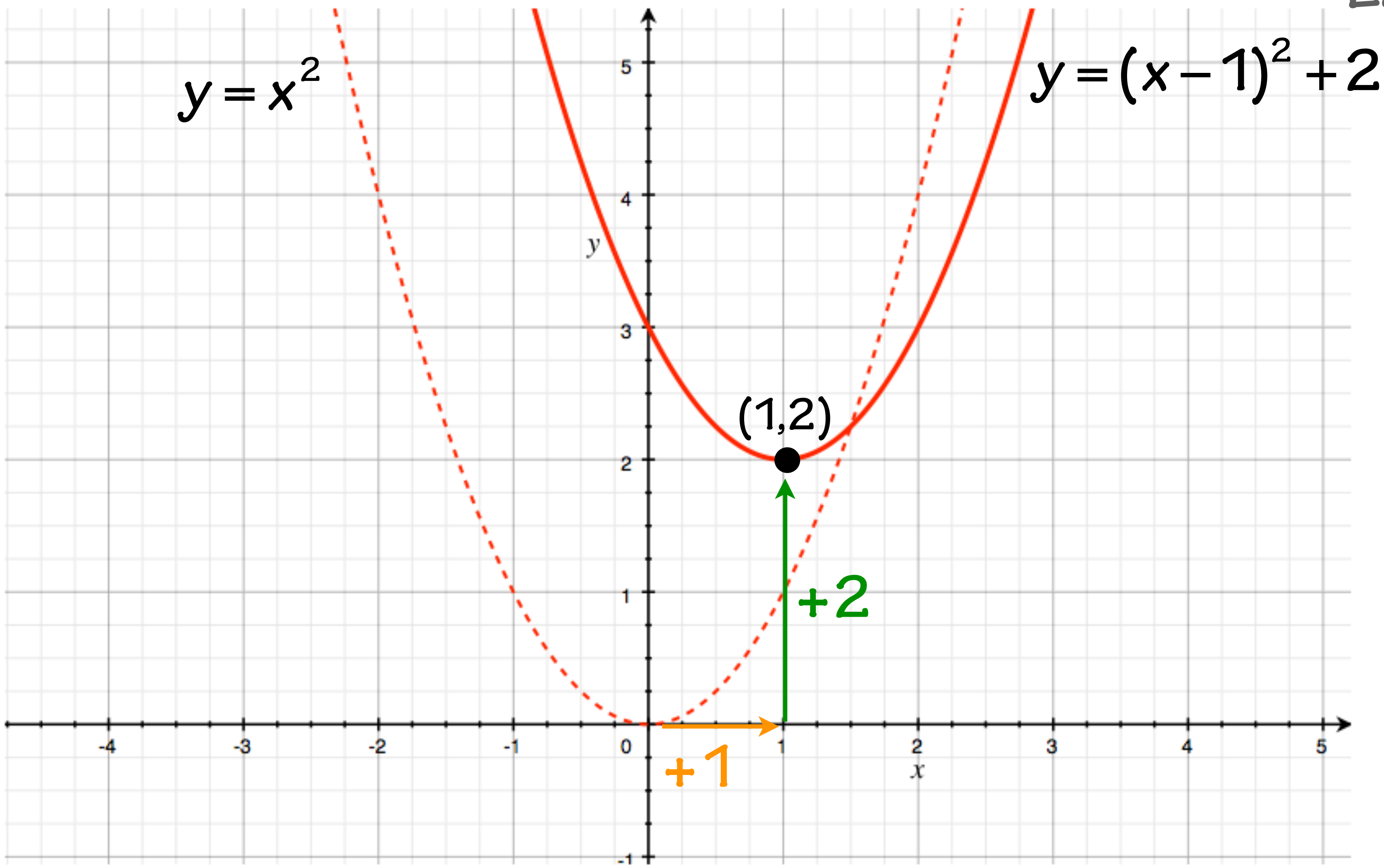
PODCAST

Transformation of functions

- Translations
- Dilations (from the x axis)
- Dilations (from the y axis)
- Reflections (in the x axis)
- Reflections (in the y axis)
- Summary
- Applying transformations
- Finding equations from transformation (graphs)
- Finding equations from transformations (from points)

Translations

- **Translations** move individual points horizontally or vertically.
- Translations can be applied to the graph of functions.
- Moving right 1 unit, up 2 units:



Original function: $y = x^2$

Each point creates an image:

$x' = x + h \qquad y' = y + k$

$x = x' - h \qquad y = y' - k$

$$y' - k = (x' - h)^2$$

$$y' = (x' - h)^2 + k$$

$$y = (x - h)^2 + k$$

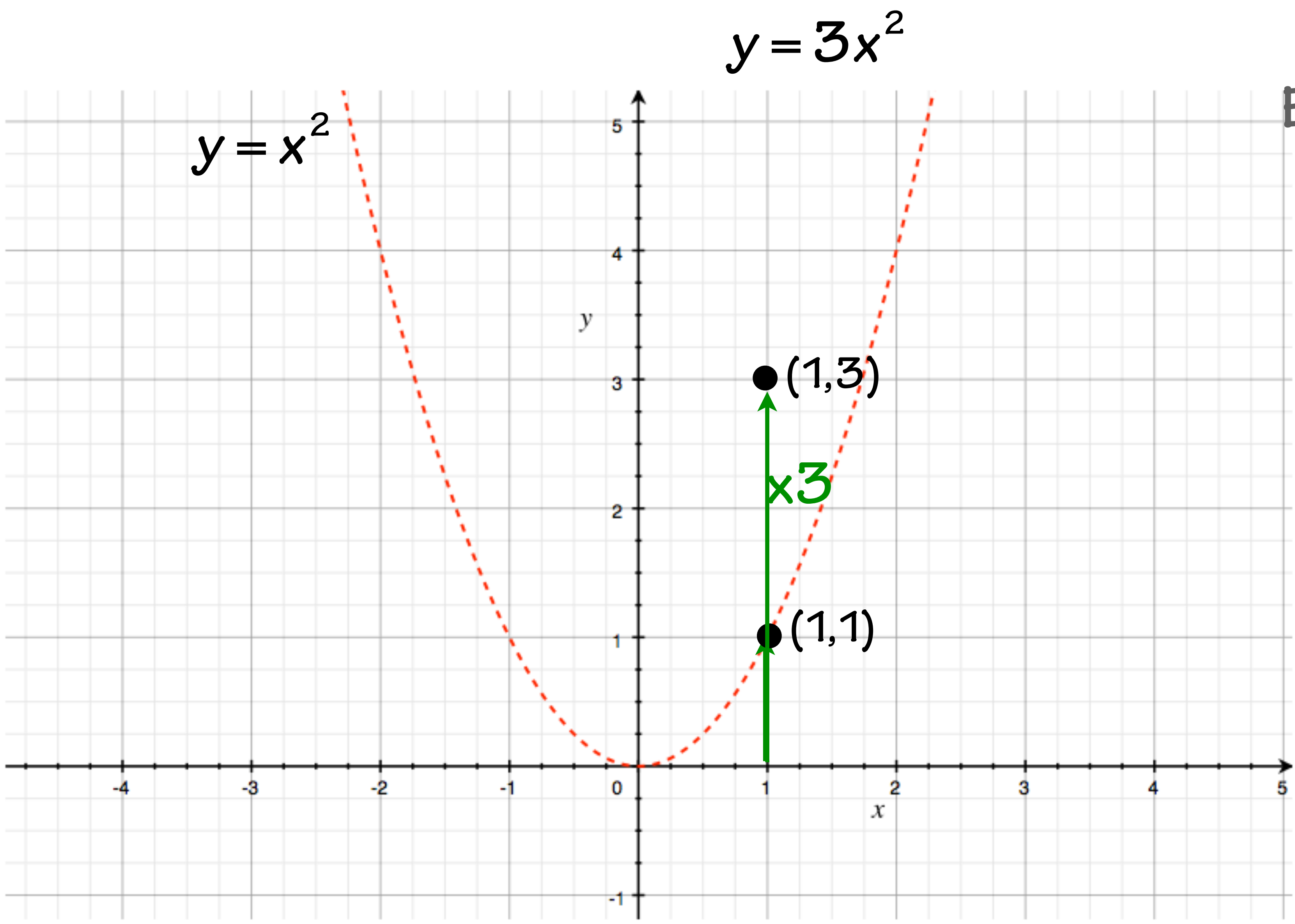
Translated function:

$$y = f(x - h) + k$$

h = horizontal translation
k = vertical translation

Dilations (from the x axis)

- **Dilations** are multiplications that stretch the graph away from an axis.
- Dilations can be from the from the x or y axis.
- A dilation of a = 3 from the x axis: stretches vertically by a factor of 3.



Original function: $y = x^2$

Each point creates an image:

$x' = x \qquad y' = a \times y$

$x = x' \qquad y = \dfrac{y'}{a}$

$$\frac{y'}{a} = (x')^2$$

$y' = a(x')^2 \qquad y = ax^2$

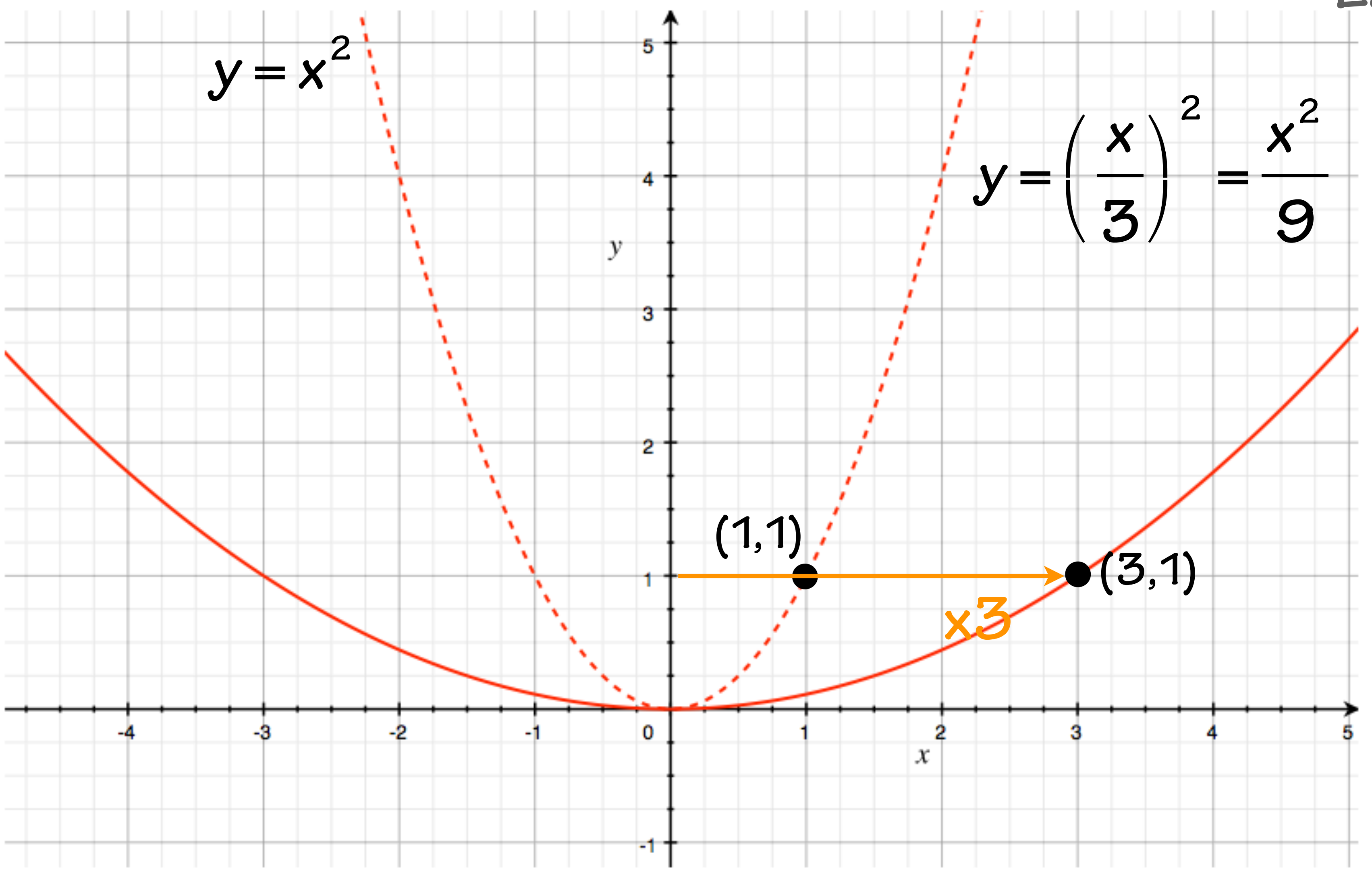
Dilated function:

$$\boxed{y = af(x)}$$

a = dilation from the x axis

Dilations (from the y axis)

- A dilation of b = 3 from the y axis: stretches horizontally by a factor of 3.



Original function: $y = x^2$

Each point creates an image:

$x' = bx$ $y' = y$

$x = \frac{x'}{b}$ $y = y'$

$y' = \left(\frac{x'}{b}\right)^2$ $y = \left(\frac{x}{b}\right)^2$

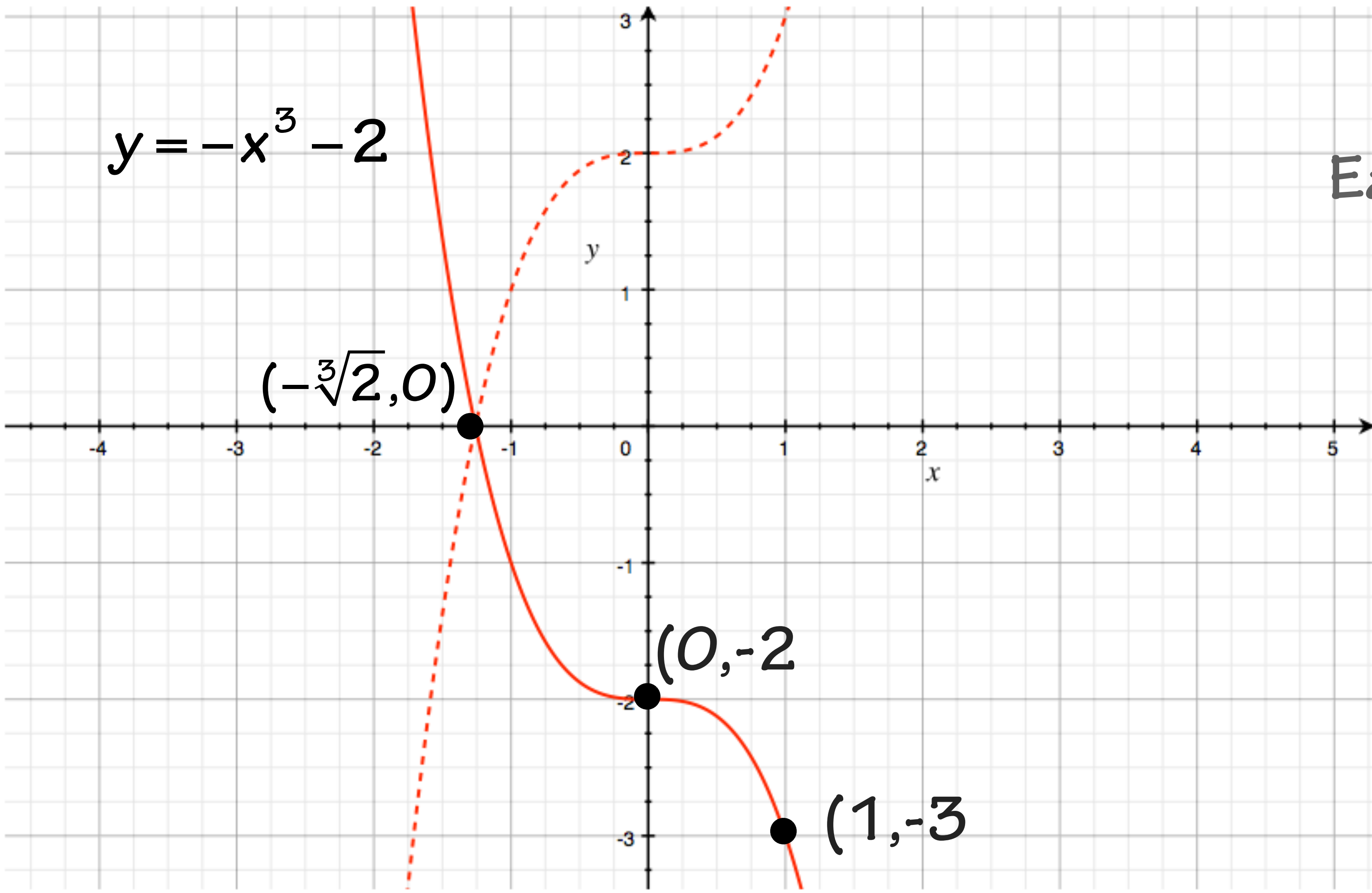
Dilated function:

$$y = f\left(\frac{x}{b}\right)$$

b = dilation from the y axis

Reflections (in the x axis)

- **Reflections** flip the graph around the x or y axis.
- Reflections keep the shape of the graph the same.
- A reflection in the x axis: signs are changed for y values.



Original function: $y=x^3+2$

Each point creates an image:

$x'=x$ $\quad$ $y'=-y$

$x=x'$ $\quad$ $y=-y'$

$$-y'=(x')^3+2$$

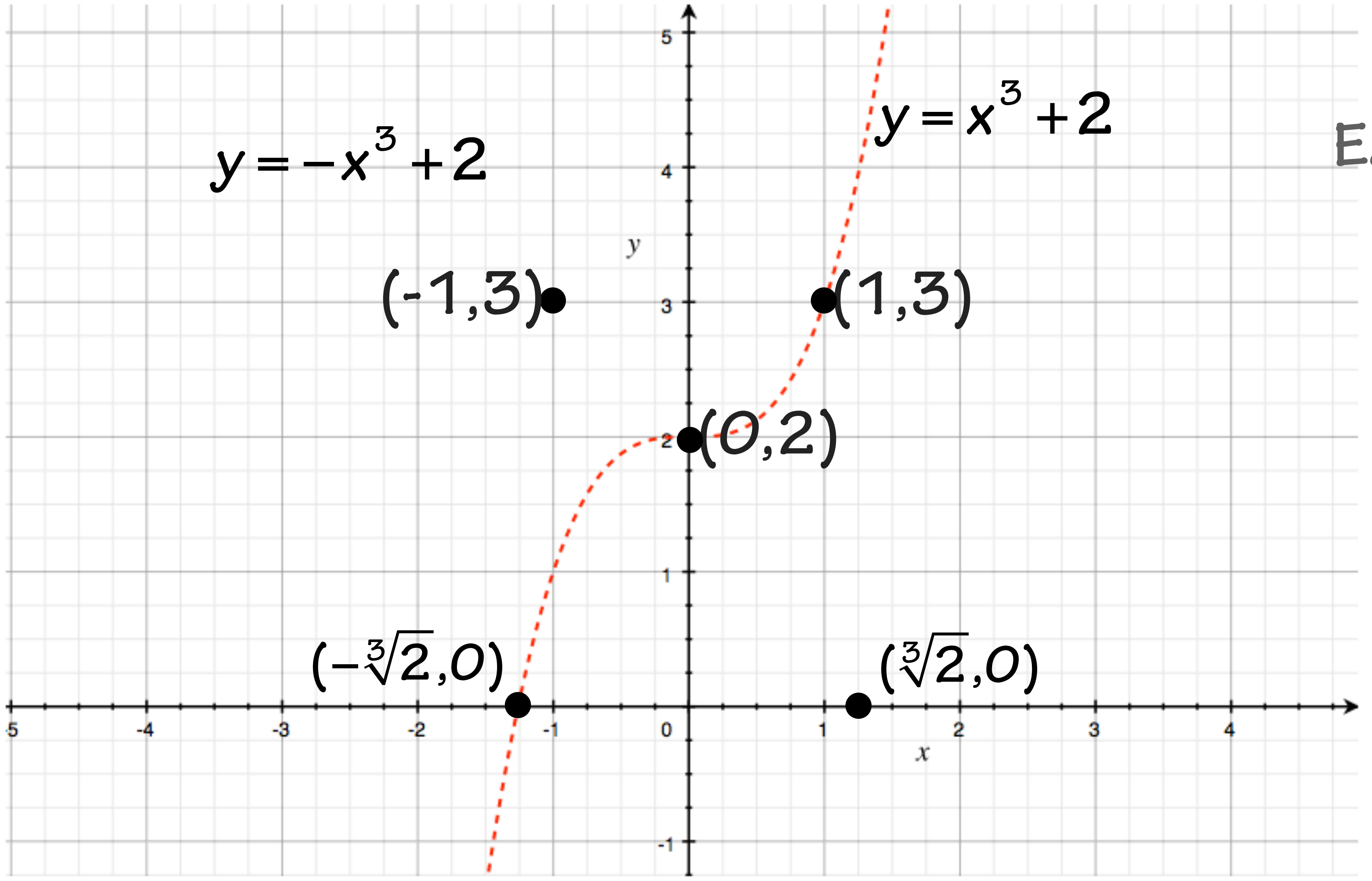
$$y'=-(x')^3-2$$

Reflected function:

$$y=-f(x)$$

Reflections (in the y axis)

- **Reflections** flip the graph around the x or y axis.
- Reflections keep the shape of the graph the same.
- A reflection in the y axis: signs are changed for x values.



Original function: $y=x^3+2$

Each point creates an image:

$x'=-x$ $\quad y'=y$

$x=-x'$ $\quad y=y'$

$$y'=(-x')^3+2$$

Reflected function:

$$\boxed{y=f(-x)}$$

Summary

Translated h units right: $y = f(x-h)$

Translated k units up: $y = f(x)+k$

Dilation by a from the x axis: $y = af(x)$

Dilation by b from the y axis: $y = f(\frac{x}{b})$

Reflection about x axis: $y = -f(x)$

Reflection about y axis: $y = f(-x)$

Translations & dilations involving x (inside the function) will always be opposites operations to y.

Applying transformations

- The order in which transformations are applied will determine the final equation.
- Transforming: $y = x^2$

1. Translation of 3 units to the right
2. Dilation by 2 from the x axis:
3. Reflection about x axis:
4. Translation of 4 units up:

$$x, y$$
$$\downarrow$$
$$x+3, y$$
$$\downarrow$$
$$x+3, 2y$$
$$\downarrow$$
$$x+3, -2y$$
$$\downarrow$$
$$x+3, -2y+4$$

$$x' = x+3 \qquad y' = -2y+4$$

$$x = x'-3 \qquad y = \frac{y'-4}{-2} = \frac{-y'+4}{2}$$

$$\frac{-y'+4}{2} = (x'-3)^2$$

$$-y'+4 = 2(x'-3)^2$$

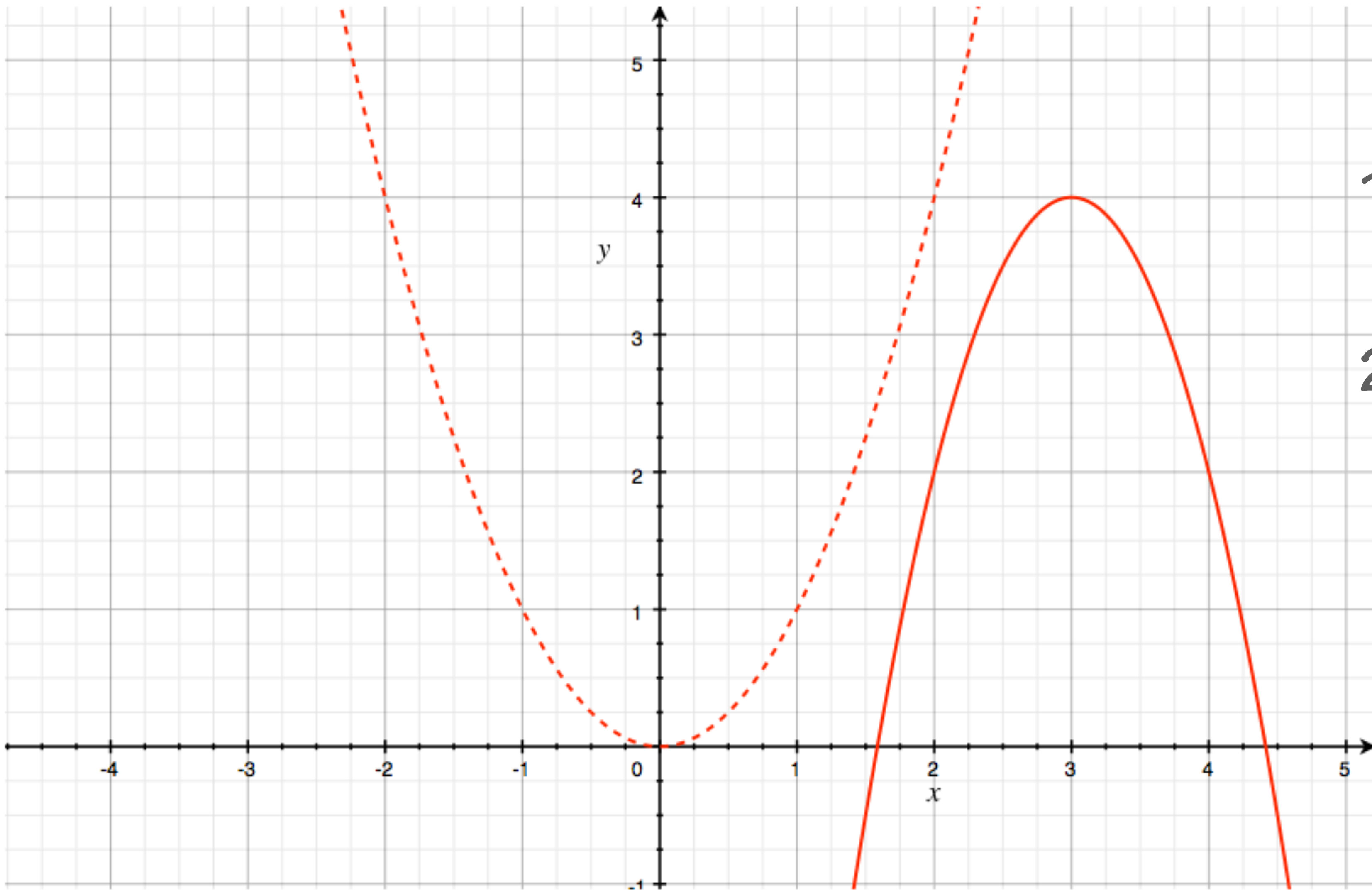
$$-y' = 2(x'-3)^2 - 4$$

$$y' = -2(x'-3)^2 + 4$$

$$y = -2(x-3)^2 + 4$$

Applying transformations: step by step

- The order in which transformations are applied will determine the final equation.

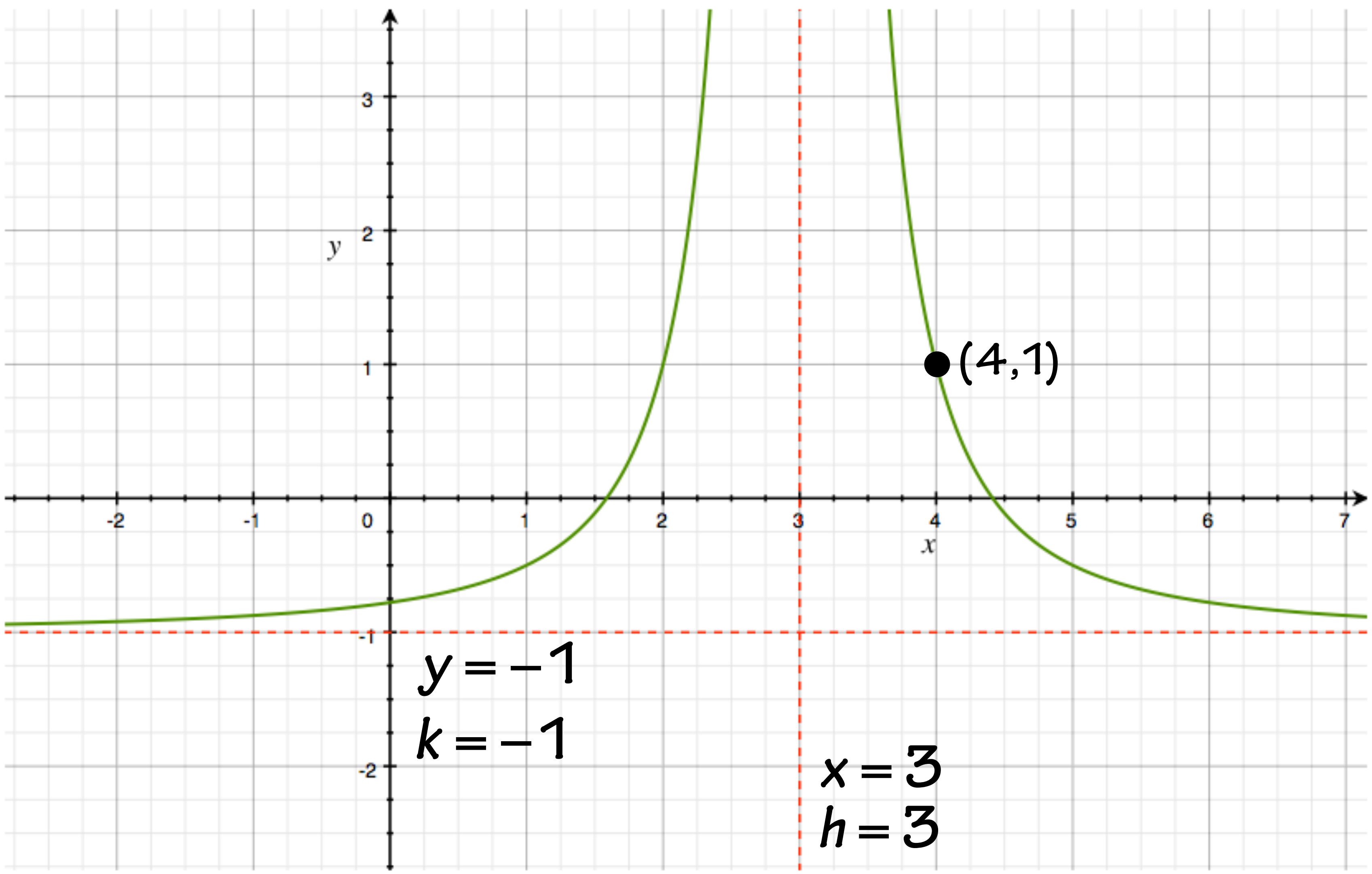


1. Translation of 3 units to the right.
2. Dilation by 2 from the x axis.
3. Reflection about x axis.
4. Translation of 4 units up.

$$y=x^2 \rightarrow y=(x-3)^2 \rightarrow y=2(x-3)^2 \rightarrow y=-2(x-3)^2 \rightarrow y=-2(x-3)^2+4$$

Finding equations from transformation (graphs)

- The equations of transformed functions can be found from graphs.
- For every unknown constant, one piece of information will be required to help to find them.
- Points, stationary points and asymptotes are used.



$$y=\frac{a}{(x-h)^2}+k$$

$$y=\frac{a}{(x-3)^2}-1$$

Substitute in a point: (4,1)

$$1=\frac{a}{(4-3)^2}-1$$

$$2=\frac{a}{(1)^2}$$

$$a=2$$

$$y=\frac{2}{(x-3)^2}-1$$

Finding equations from transformations (from points)

- The equations of transformed functions can be found from points.
- For every unknown constant one piece of information will be required to help to find them.
- Simultaneous equations are used to find the unknowns from points.

An equation of the form: $y = \sqrt{ax+b}$

Passes through the points: $(7,6)$ and $(3,4)$

$$4 = \sqrt{3a+b}$$
$$6 = \sqrt{7a+b}$$

$$16 = 3a+b$$
$$36 = 7a+b$$

$$20 = 4a$$
$$a = 5$$

$$16 = 3a+b$$
$$16 - 3\times 5 = b$$
$$b = 1$$

$$y = \sqrt{5x+1}$$

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