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MATHS METHODS

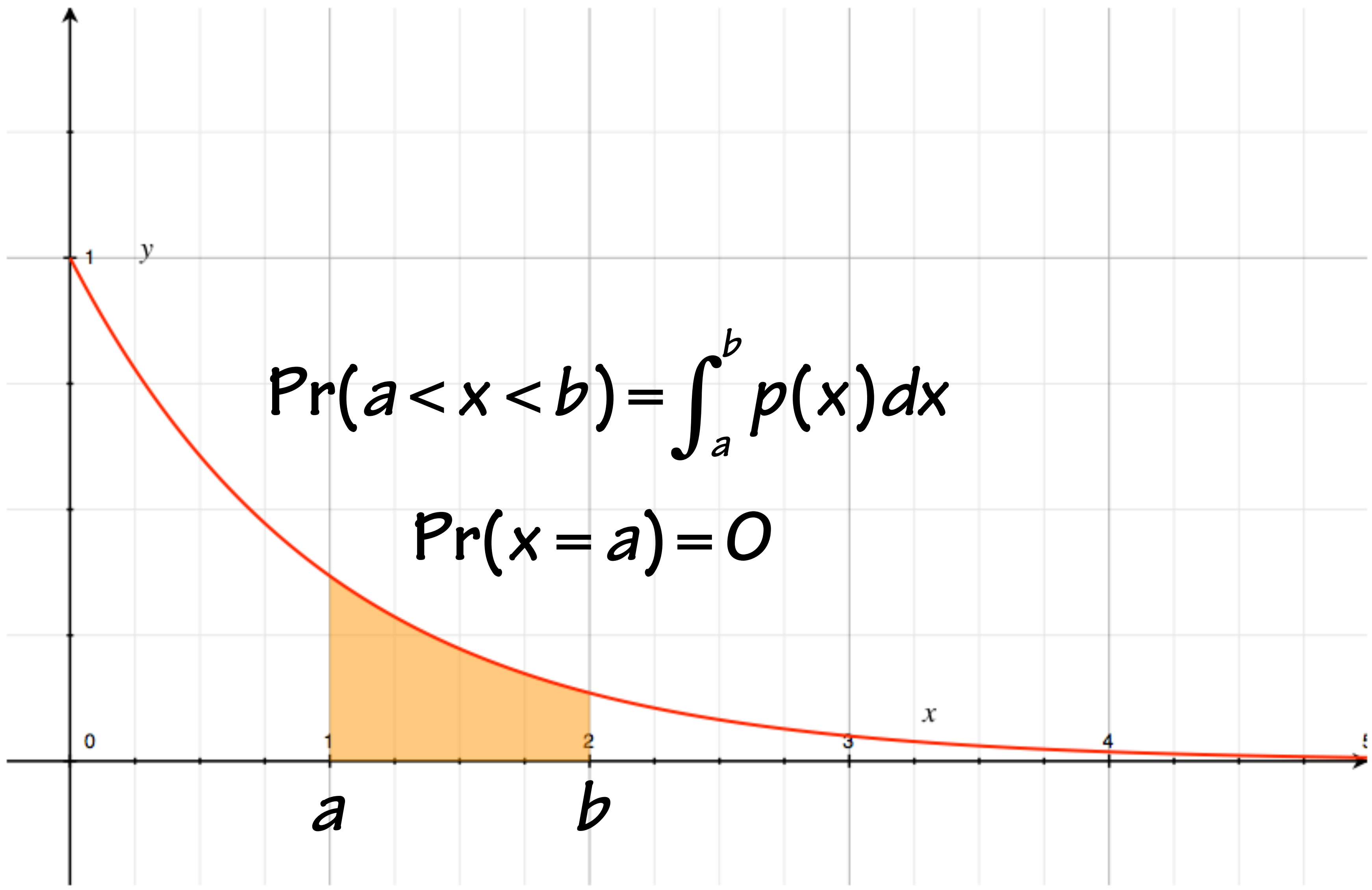
PODCAST

Continuous probability distributions

- Probability density functions
- Calculating probabilities
- Median value
- Mean value
- Variance
- Cumulative probability distributions

Probability density functions

- Continuous variables (such as measurements of time, length etc) can be described by **probability density functions**.
- The chance of a measurement **between** two values (a & b) for a probability density function p(x) is represented by the area under the graph:



$$Pr(a < x < b) = \int_a^b p(x)\,dx$$

$$Pr(x = a) = 0$$

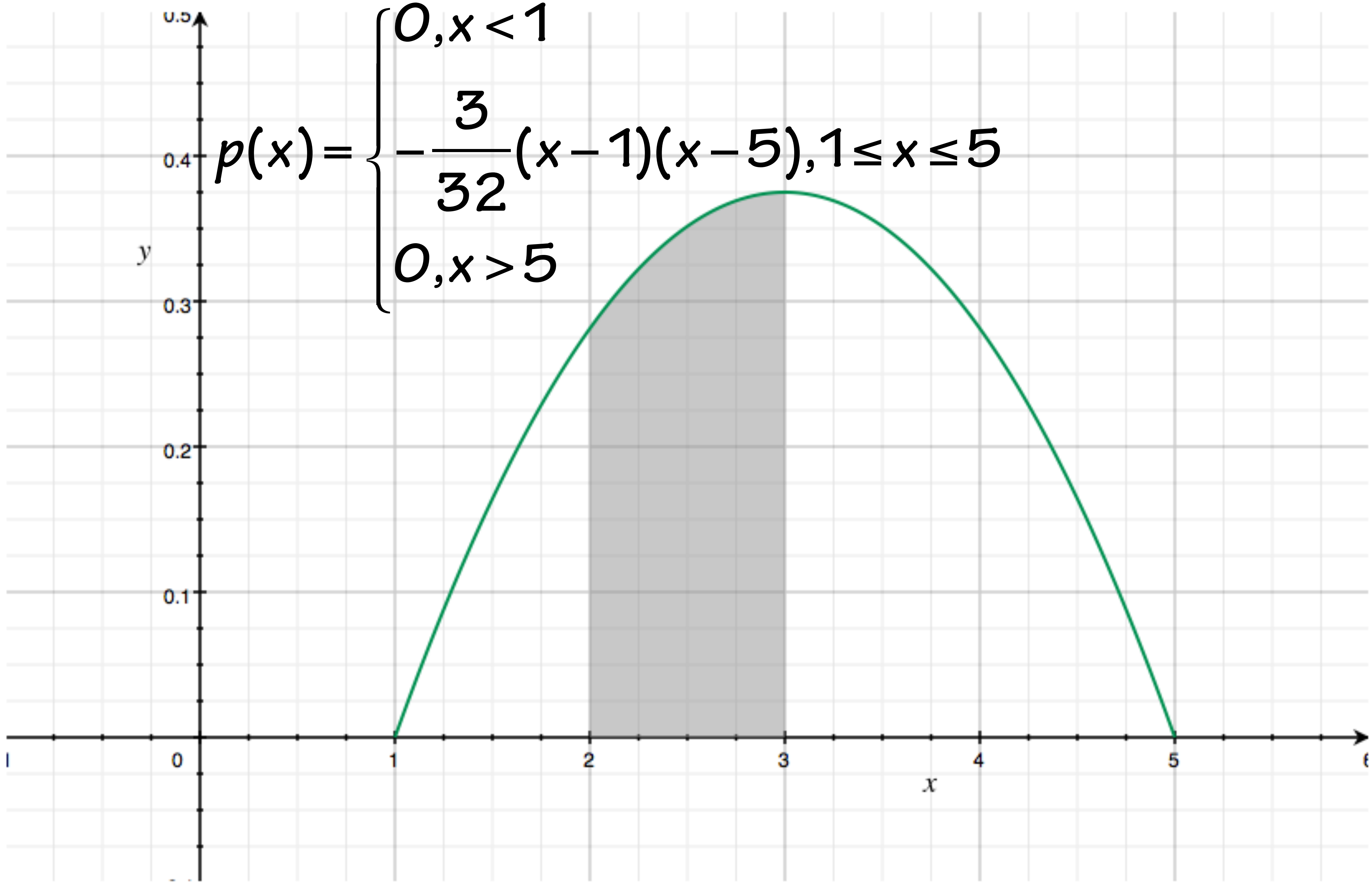
Probability density functions

- For a function p(x) to describe a probability density function:
- p(x) is continuous function within the domain of the possible values of x.
- The area under the curve between the lowest & highest possible values of x is exactly 1.
- All values of p(x) are positive or zero; not negative.
- Most probability density functions are described by a hybrid of two or more functions over the domain of R.

Calculating probabilities

- Find the probability that is x is between 2 and 3.

$$p(x) = \begin{cases} 0, x < 1 \\ -\dfrac{3}{32}(x-1)(x-5), 1 \le x \le 5 \\ 0, x > 5 \end{cases}$$



Calculating probabilities

$$\Pr(a<x<b)=\int_a^b p(x)\,dx \qquad p(x)=-\frac{3}{32}(x-1)(x-5)$$

$$\Pr(2<x<3)=\int_2^3 -\frac{3}{32}(x-1)(x-5)\,dx$$

$$\Pr(2<x<3)=\int_2^3 -\frac{3}{32}(x^2-6x+5)\,dx$$

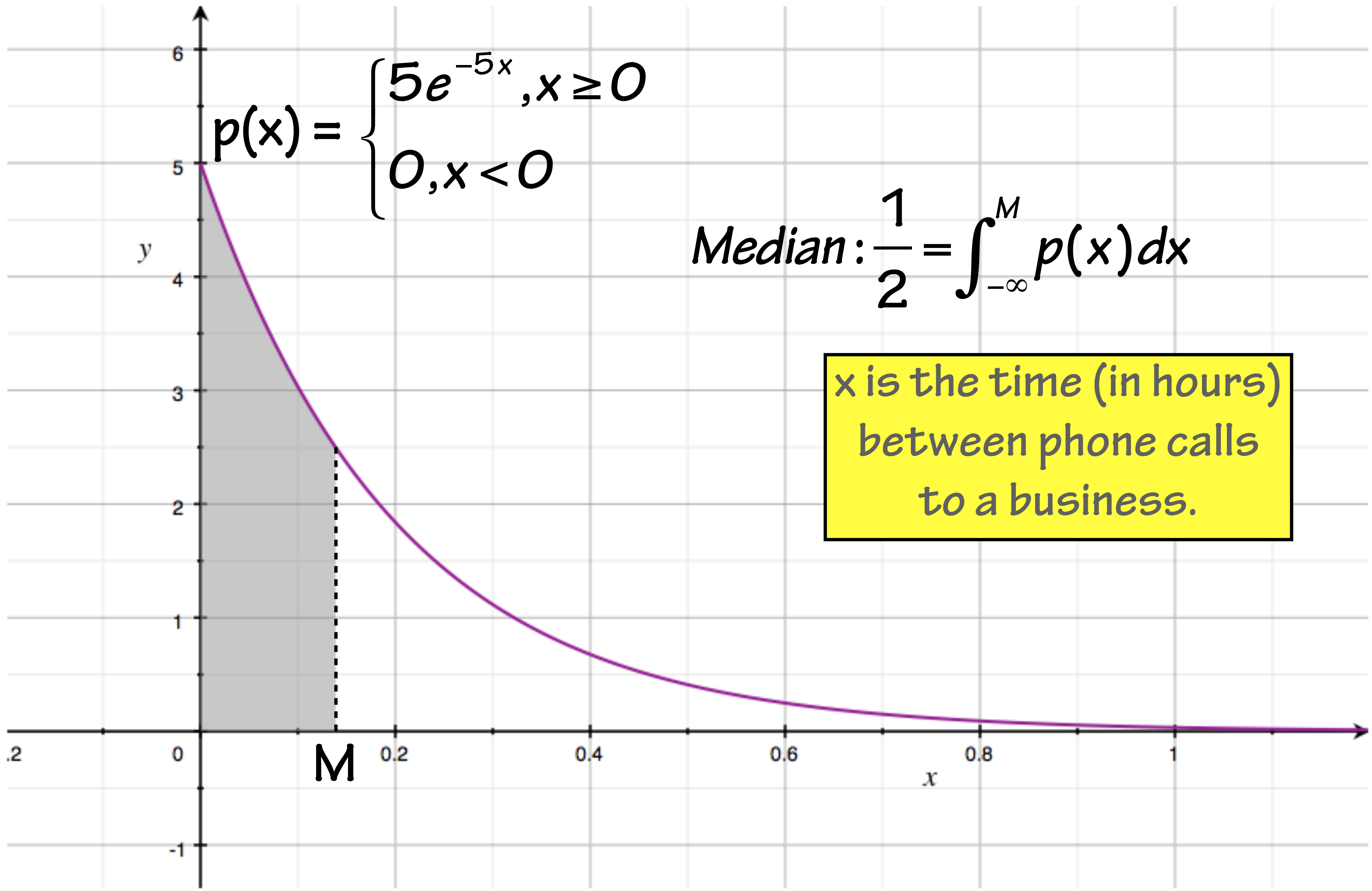
$$\Pr(2<x<3)=-\frac{3}{32}\left[\frac{x^3}{3}-3x^2+5x\right]_2^3$$

$$\Pr(2<x<3)=-\frac{3}{32}\left[\left(\frac{(3)^3}{3}-3(3)^2+5(3)\right)-\left(\frac{(2)^3}{3}-3(2)^2+5(2)\right)\right]$$

$$\Pr(2<x<3)=-\frac{3}{32}\left(9-27+15-\frac{8}{3}+12-10\right)=\frac{11}{32}\approx 0.344$$

Median value

- The median value (M) of a continuous probability distribution p(x) is the value of x which is the centre of the area under the graph of p(x).



$$p(x) = \begin{cases} 5e^{-5x}, x \geq 0 \\ 0, x < 0 \end{cases}$$

$$Median: \frac{1}{2} = \int_{-\infty}^{M} p(x)\,dx$$

x is the time (in hours) between phone calls to a business.

Calculating median value

$$\frac{1}{2}=\int_{-\infty}^{m}p(x)\,dx \quad p(x)=5e^{-5x}, x\geq 0$$

$$\frac{1}{2}=\int_{0}^{m}5e^{-5x}\,dx$$

$$\frac{1}{2}=\left[-e^{-5x}\right]_{0}^{m}$$

$$\frac{1}{2}=-e^{-5m}--e^{0}$$

$$\frac{1}{2}=-e^{-5m}+1$$

$$-\frac{1}{2}=-e^{-5m}$$

$$\frac{1}{2}=e^{-5m}$$

$$\log_e\left(\frac{1}{2}\right)=-5m$$

$$-\log_e\left(\frac{1}{2}\right)=5m$$

$$\log_e(2)=5m$$

$$m=\frac{\log_e(2)}{5}$$

$$m\approx 0.138$$

Half the time there is less than 0.138 hours (8.3 minutes) between calls.

Mean value

- The mean value of a discrete probability distribution is found by adding together the products of values and probabilities.
- For a continuous distribution, this is calculated by integrating the product of x and p(x).

$$E(X) = \sum x \, Pr(X = x)$$

Discrete distribution

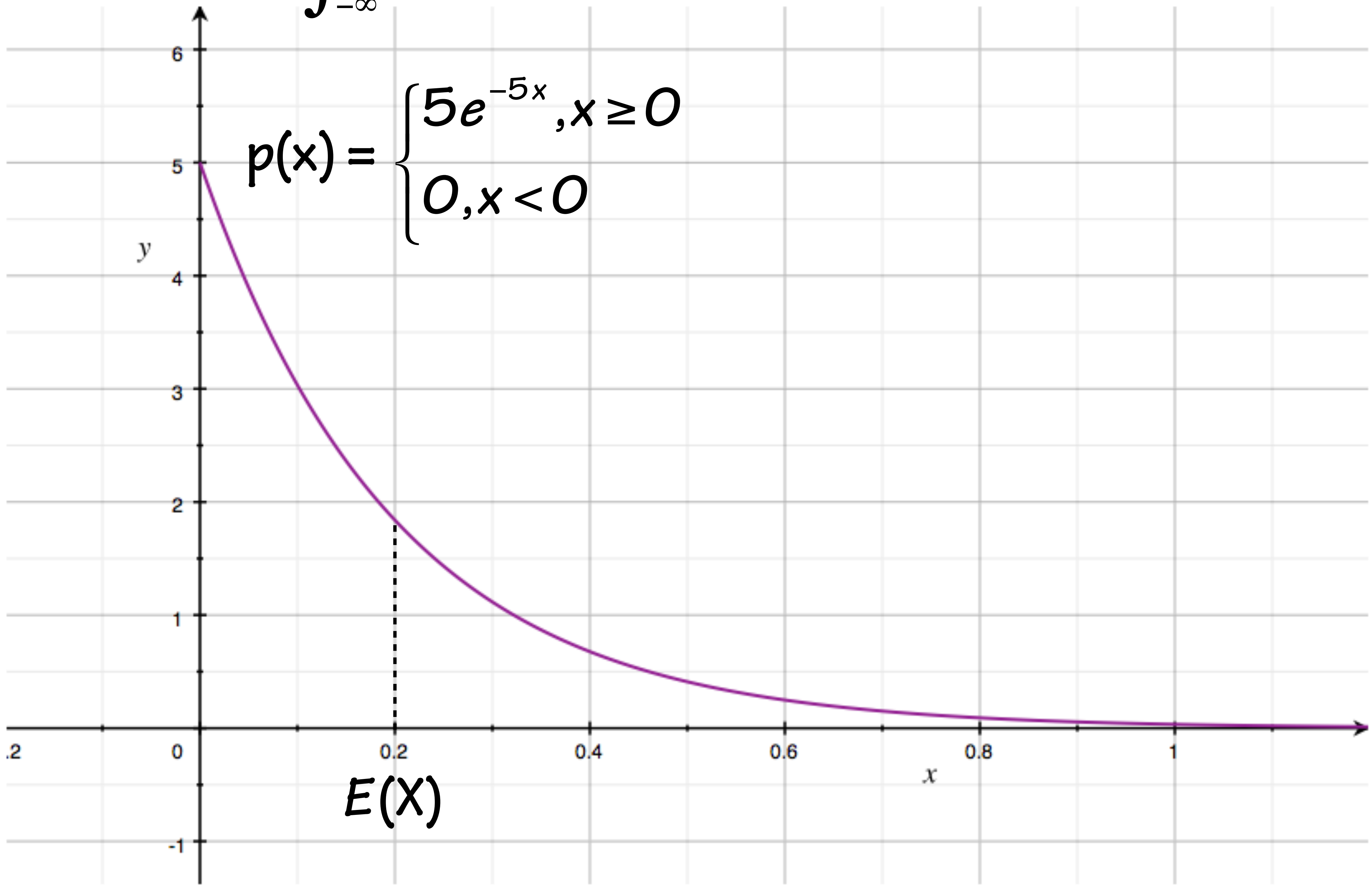
$$Mean = E(x) : \int_{-\infty}^{\infty} x \, p(x) \, dx$$

Continuous distribution

Calculating mean value

$$Mean = E(X) : \int_{-\infty}^{\infty} x\,p(x)\,dx$$

$$p(x) = \begin{cases} 5e^{-5x}, x \geq 0 \\ 0, x < 0 \end{cases}$$



Calculating mean value

$$Mean = E(x): \int_{-\infty}^{\infty} x\,p(x)\,dx \qquad p(x) = 5e^{-5x}, x \geq 0$$

$$E(X): \int_{0}^{\infty} x5e^{-5x}\,dx$$

Not part of the Maths Methods course!
(But it can be solved on a CAS calculator.)

$E(X) = 0.20$

The average time between calls is 0.2 hours. (12 minutes)

Variance

- The **variance** of a probability distribution is the expected value of the difference between x values and the mean.
- It can also be calculated from the subtracting the square of the expected value of X from the expected value of the squares of X.
- The standard deviation is the most commonly used measure of spread.
- A normally distributed probably density function has 68% of outcomes within one standard deviation of the mean and 95% within two.

$$\text{Var(X)} = E(X - \mu^2)$$

$$\text{Var(X)} = E(X^2) - (E(X))^2$$

$$\text{Standard deviation (X)} = \sqrt{Var(x)}$$

Calculating variance

$$Var(X) = E(X^2) - E(X)^2$$

$$p(x) = \begin{cases} 5e^{-5x}, x \geq 0 \\ 0, x < 0 \end{cases}$$

$$E(X) = 0.20$$

$$E(X)^2 = 0.04$$

$$E(X^2) = \int_0^{\infty} x^2 5e^{-5x}\,dx$$

$$E(X^2) = 0.08$$

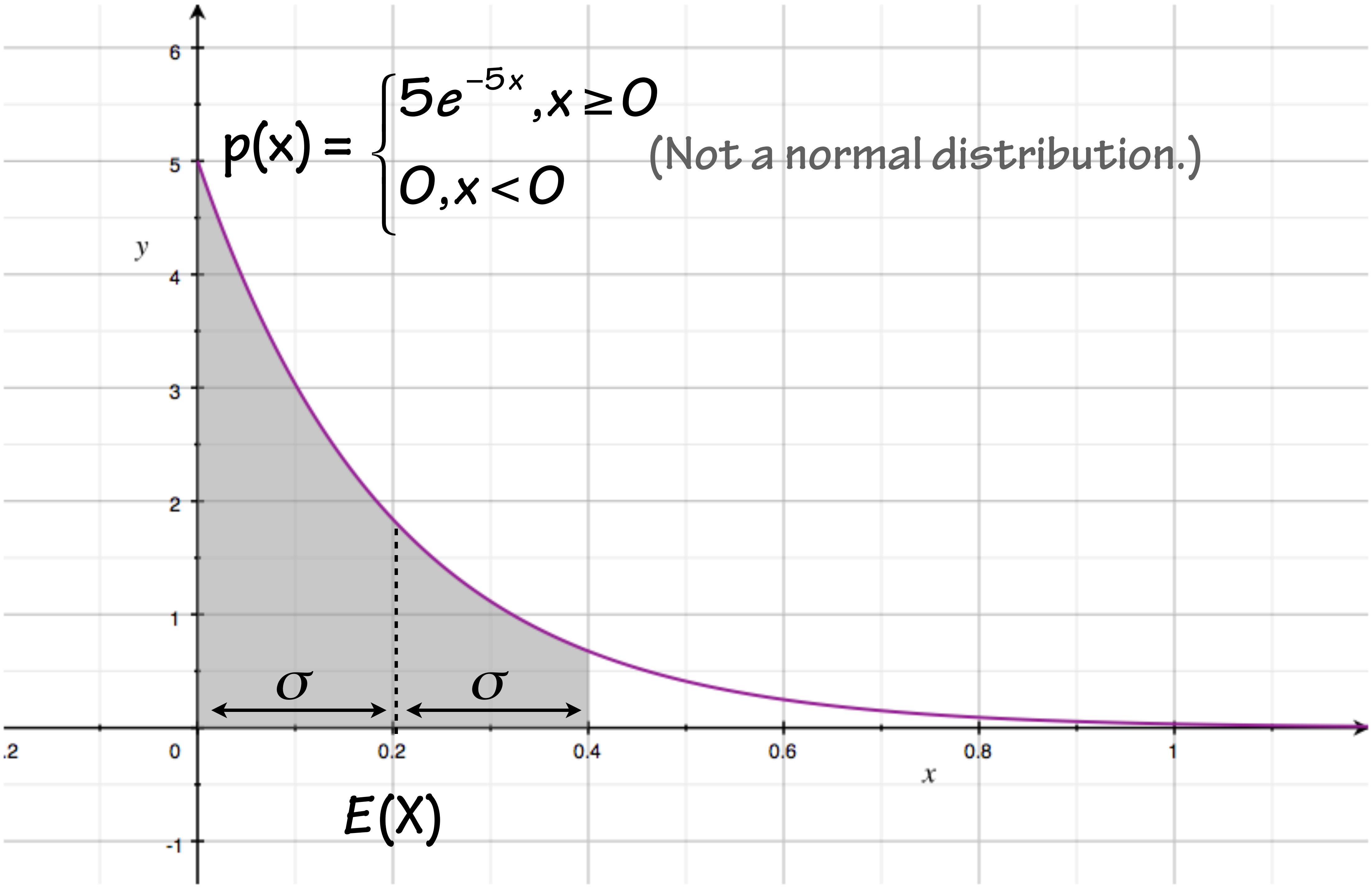
$$Var(X) = E(X^2) - (E(X))^2$$

$$Var(X) = 0.08 - 0.04$$

$$Var(X) = 0.04$$

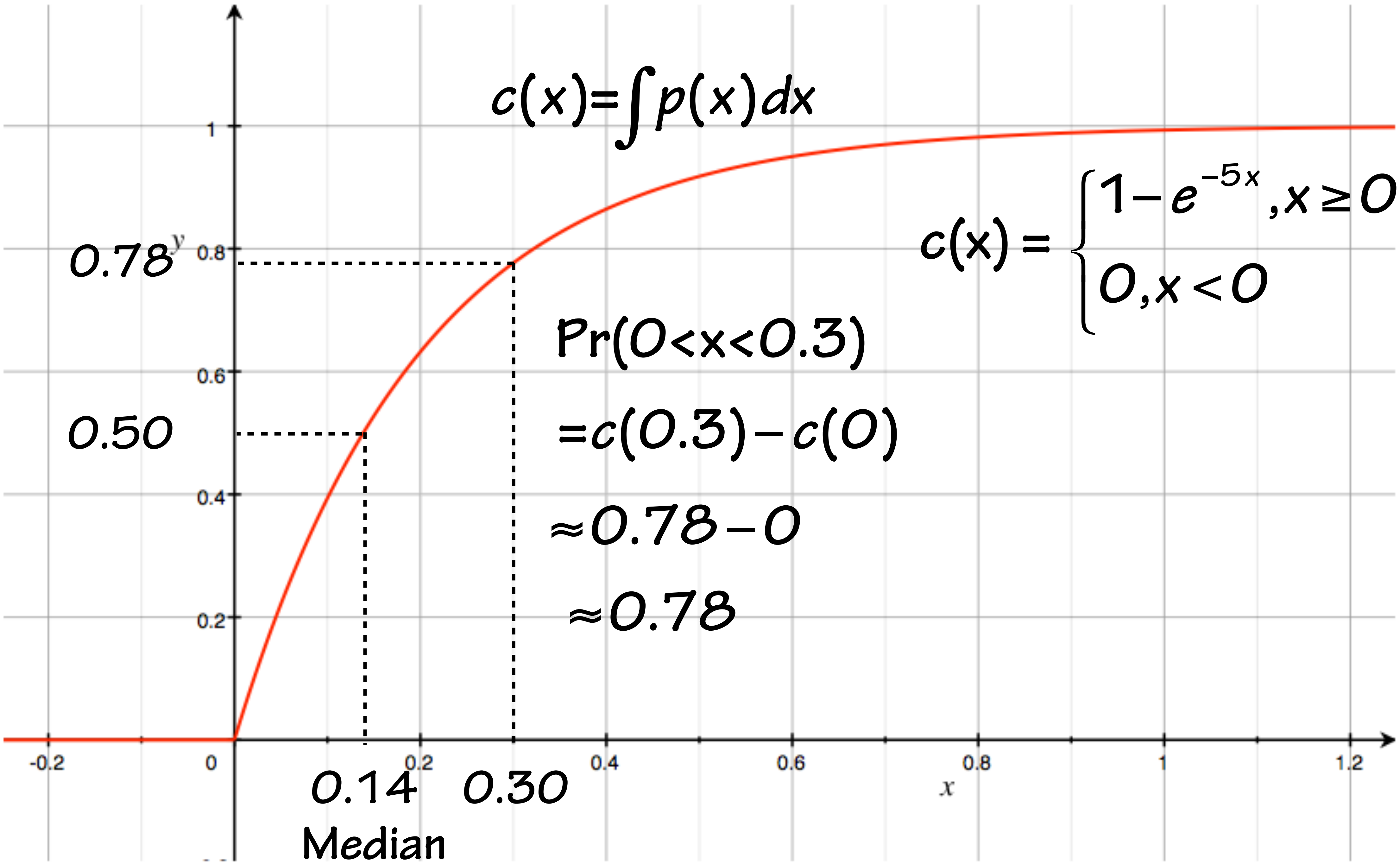
$$\text{Standard deviation }(X) = \sqrt{0.04} = 0.2$$

Calculating variance



Cumulative probability distributions

- The cumulative probability distribution c(x)is found by integrating p(x).
- Probabilities are read from the vertical axis.
- Quartile values can be read from the horizontal axis.

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